

Description**Method and arrangement for designing a technical system**

- 5 The invention relates to a method and an arrangement for designing a technical system.

In order to design a complex technical system it is often necessary to optimize said system in respect of a plurality of
10 contradictory criteria. Said criteria influence target functions of the system, such as, for example, manufacturing costs or efficiency. In addition, possible operating points of the system can be restricted by auxiliary conditions. This leads to the problem of determining a set of optimal operating
15 points for the system, that is to say the set of possible operating points of the system with which it is not possible to optimize said operating points further simultaneously with regard to all criteria. From said set of optimal points individual users can then select the most suitable operating
20 points of the system for their applications while taking into account secret criteria or expert knowledge.

A weighting method for optimizing technical systems in respect of a plurality of criteria is known from [1], wherein scaling
25 parameters are employed to apply transformations to scalar-value optimization problems. Said method has the disadvantage that it is numerically very involved, because very many scalar-value optimizations have to be performed. Furthermore, the selection and variation of the scaling parameters
30 necessitates an interaction with a user and in this respect cannot be automated.

A stochastic method for optimizing technical systems in respect of a plurality of criteria, wherein a stochastic
35 differential equation is used to solve the optimization

problem, is described in [2]. Said method has the disadvantage that it is very involved in numerical terms, because a multiplicity of quadratic optimization problems have to be solved. A further disadvantage lies in the fact that with said
5 method the individual target functions are not weighted, as a result of which important information for selecting an optimal point is not available to the user.

A homotopy method for optimizing technical systems in respect
10 of multiple criteria, wherein in addition to weighting factors for the target functions Lagrange multipliers are used in order to take auxiliary conditions into account, is known from [3]. The disadvantage of said method lies in the fact that an interaction with the user is necessary and in this respect
15 said method cannot be automated.

The **object** of the invention is therefore to create an automated and numerically efficient method for designing a technical system.

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This object is achieved according to the features of the independent claims. Developments of the invention will also be apparent from the dependent claims.

25 With the method according to the invention for designing a technical system, the technical system is modeled by means of a predetermined set of target functions which are dependent on parameters. In this modeling process each individual target function is weighted with a weighting factor. The method
30 solves an equation system comprising the parameters and the weighting factors as variables in a variable space, with solutions of the equation system forming operating points of a solution space in the variable space. In the method according to the invention the operating points are determined by means
35 of a predictor-corrector method, according to which, starting

from a first operating point, a predictor generated by means of a stochastic variable is determined in the variable space, and subsequently, in a corrector step, a second operating point is determined. Said determined operating points are used
5 here to design the technical system. The inventive method can be used to design a new technical system or modify or, as the case may be, adapt an existing technical system.

An advantage of the invention resides in the fact that the
10 method is automated through the generation of the predictor by means of a stochastic variable and so there is no longer any need for an intervention on the part of the user. The linking of the numerical predictor-corrector method with stochastic methods guarantees efficient use of the computer resources for
15 calculating operating points of a technical system.

In an advantageous embodiment of the invention the predictor is predetermined by means of random numbers, so that in particular during the execution of the method a random number
20 generator can be used and by this means the automation of the method is ensured in a simple manner.

In a further particularly advantageous embodiment of the invention the random numbers are normally distributed. What
25 this achieves is that the trajectory of operating points which forms in the solution space during the execution of the method ensures a uniform distribution in the entire solution space and so ensures good coverage of all possible operating points. As a result of the use of normally distributed random numbers,
30 in particular a Brown movement on the solution space can be modeled by means of the method according to the invention.

Preferably the operating points which are determined by means of the method according to the invention are what are known as
35 pareto-optimal points which cannot be optimized further in

relation to all target functions. In the method according to the invention, in particular the points with positive weighting factors in the solution space are selected as operating points.

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In a further advantageous embodiment of the invention the operating points must also satisfy one or more auxiliary conditions, with the or each auxiliary condition being represented by a further variable of the equation system in the variable space. In this case the auxiliary conditions can be equality auxiliary conditions and/or inequality auxiliary conditions. With inequality auxiliary conditions a slack variable is preferably introduced, by means of which the inequality auxiliary conditions can be transformed into equality auxiliary conditions. The use of slack variables will be explained in more detail in the detailed description of an exemplary embodiment.

The solution space of the operating points is preferably a manifold, in particular a submanifold in the variable space. In [3] it is explained under what preconditions the solution space forms such a manifold.

Since in particular at the start of the method according to the invention a first operating point is present initially, in a special embodiment of the invention this first valid operating point is determined by means of a weighting method, the use of weighting methods already being known from the prior art (see [1]).

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With the predictor-corrector method used in the invention, a tangential plane to the solution space is determined, preferably in the first operating point, and the predictor is then specified in said tangential plane.

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In a development of the method according to the invention, if a negative predictor with one or more negative weighting factors occurs, a new predictor is determined by a reflection at a subplane of the solution space of the valid operating points. By this means new regions of valid operating points can be determined, which operating points can be of particular relevance to the user in terms of secret supplementary criteria or his/her expert knowledge.

10 In a preferred embodiment of the invention, in the reflection step a point of intersection of the trajectory that runs between the first operating point and the negative predictor with a subplane of the solution space is determined. The tangential component of the vector spanned by the point of
15 intersection and the negative predictor to the relevant subplane of the solution space is then determined, with those weighting factors which were negative for the negative predictor in the points of the subplane now being equal to zero. Next, the normal component, associated with said
20 tangential component, of the vector spanned by the point of intersection and the negative predictor is determined. Finally, the new predictor is determined by two times subtraction of the normal component from the negative predictor.

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A Newton method known from the prior art, which method is easily convertible numerically, is preferably used for the corrector method employed in the invention.

30 The operating points are preferably determined by means of iterations of the predictor-corrector method, with the second operating point of the preceding iteration step being used in a current iteration step as the first operating point of the predictor-corrector method. In this case the method is
35 terminated by means of, for example, an abort condition. In an

advantageous embodiment of the invention the abort condition is met when a predetermined number of operating points has been determined and/or a predetermined time limit has been reached.

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In addition to the above-described method for designing a technical system, the invention also relates to an arrangement for designing a technical system by means of which the above-described method can be performed. In particular the method
10 comprises a processor unit by means of which it is made possible for the predictor to be generated using a stochastic variable.

The arrangement preferably comprises a random number generator
15 for generating random numbers which represent the stochastic variable.

The invention or any development thereof described in the foregoing can also be implemented by means of a computer
20 program product which has a storage medium on which is stored a computer program which is executable on a computer and executes the invention.

Exemplary embodiments of the invention are described and
25 explained in the following with reference to the drawings, in which:

Fig. 1 shows a flowchart of the method according to the invention for designing a technical system;

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Fig. 2 shows a diagram which illustrates the predictor-corrector method used in the invention;

Fig. 3 shows a diagram which illustrates the reflection
35 method used in a variation of the invention, and

Fig. 4 shows a processor unit for performing the method according to the invention.

- 5 A flowchart of the method according to the invention for designing a technical system is shown in Fig. 1.

First, in step 101, a description form of the technical system is selected. Said description form comprises a predetermined
 10 number k of target functions $\underline{f}=(f_1, \dots, f_k)$, with each of the target functions being dependent on n predetermined parameters x_1 to x_n of the technical system. The target functions are for example the investment costs f_1 and the efficiency f_2 of the technical system. In this case the target functions are
 15 described by means of the following equation:

$$\underline{f}(\underline{x}) = \left(\frac{f_1(\underline{x})}{f_2(\underline{x})} \right) = \left(\frac{\text{Investment.costs}(\underline{x})}{\text{Efficiency}(\underline{x})} \right)$$

where $\underline{x}=(x_1, \dots, x_n)$.

- 20 The parameters x_1 to x_n can be configuration parameters or operating parameters of the technical system.

By means of the method according to the invention the valid operating points which are used for the design of the
 25 technical system are determined by the optimization of the target functions in respect of the parameters, whereby not all target functions f_1 to f_k can be optimized simultaneously since the optimization criteria are generally in competition with one another.

The technical system is further limited in the valid operating points by a predetermined number m of auxiliary conditions $\underline{h}=(h_1(\underline{x})\dots,h_m(\underline{x}))$ which can be expressed by means of the following equation:

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$$\underline{h}(\underline{x})=\underline{0}$$

where $\underline{0}=(0,\dots,0)$ represents a zero vector. In this case what is involved is an equality auxiliary condition, with an
 10 inequality auxiliary condition also being able to be considered as an alternative. An inequality condition of this kind is, for example:

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$$\underline{h}(\underline{x})<\underline{0} \quad \text{or} \quad \underline{h}(\underline{x})>\underline{0} \quad .$$

In order to solve the optimization problems by means of inequality auxiliary conditions, a number m of slack variables $\underline{s}=(s_1,\dots,s_m)$ are introduced, by means of which the inequality auxiliary conditions can be transformed into the following
 20 equality auxiliary conditions:

$$\underline{h}(\underline{x})+\underline{s}=\underline{0} \quad \text{or} \quad \underline{h}(\underline{x})-\underline{s}=\underline{0} \quad .$$

With the optimization method described in the present
 25 embodiment of the invention, the valid operating points are known as pareto-optimal points, which satisfy the following condition:

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$$\min_{\underline{x} \in R} \{ \underline{f}(\underline{x}) \}, R = \{ \underline{x} \in \mathbb{R}^n \mid \underline{h}(\underline{x}) = 0 \}$$

It can be shown that the solutions of this optimization problem are the solutions of the following nonlinear equation systems:

$$F(\underline{x}, \underline{\lambda}, \underline{\alpha}) = \begin{pmatrix} \sum_{i=1}^k \alpha_i \cdot \nabla f_i(\underline{x}) + \sum_{j=1}^m \lambda_j \cdot \nabla h_j(\underline{x}) \\ \underline{h}(\underline{x}) \\ \sum_{i=1}^k \alpha_i - 1 \end{pmatrix} = \underline{0}$$

In this case the auxiliary conditions are taken into account
 5 by the Lagrange multipliers $\underline{\lambda} = (\lambda_1, \dots, \lambda_m)$, and the target
 functions f_i are weighted with weighting factors α_i , whereby
 care must be taken to ensure that the total of all weighting
 factors is normalized to one, i.e. $\sum_{i=1}^k \alpha_i - 1 = 0$. In this case,
 in particular, the individual weighting factors α_i can also be
 10 negative or equal to zero. The solutions of the optimization
 problem are therefore vectors $(\underline{x}, \underline{\lambda}, \underline{\alpha})$ in the $(n+m+k)$ -
 dimensional variable space of the above equation system.

As shown in [3], under certain conditions the solutions of
 15 this equation system describe a $(k-1)$ -dimensional submanifold
 M in the variable space.

The below described numerical steps for determining valid
 operating points are essentially based on the homotopy method
 20 described in [3], wherein a predictor-corrector method is used
 for determining pareto-optimal points.

In step 102, proceeding from the description form 101 of the
 technical system, a first pareto-optimal point z is determined
 25 by means of a standard method such as, for example, the
 weighting method.

In this first pareto-optimal point, in the next step 103, a
 $(k-1)$ -dimensional tangential plane $T_z M$ to the manifold M of
 30 the valid operating points is determined in point z . Toward
 that end, a Jacobi matrix of the equation system F in point z

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is subjected to a QR factorizing. From this, an orthonormal basis $\{ \mathbf{q}_1 \dots \mathbf{q}_{k-1} \}$ is then determined which spans the tangential plane. The individual numerical steps performed in this process are described in detail in [3].

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In the next step 104, a predictor \mathbf{y} is determined in this tangential plane, with the predictor - in contrast to the homotopy method described in [3] - being generated by means of a normally distributed random number vector \mathbf{b} of the dimension $k-1$ in the tangential plane. In this case the predictor \mathbf{y} has the following form:

$$\mathbf{y} = \mathbf{z} + (\mathbf{q}_1 \dots \mathbf{q}_{k-1}) \mathbf{b}$$

15 Through the use of a random number vector such as this, a Brown movement can be modeled on the submanifold M , with the Brown movement being able to be represented approximately as follows:

$$20 \quad dZ_t = \varepsilon P(Z_t) dB_t$$

where

25 $P(\mathbf{z})$ is a projection matrix onto the tangential plane $T_{\mathbf{z}}M$ in the valid operating point \mathbf{z} ,

ε is a scaling factor, and

$B_t, t \in \mathbb{R}_0^+$ is a Brown movement in the variable space.

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In order to model this movement, the $k-1$ -dimensional normal distribution $N(0_{k-1}, t\Delta\varepsilon I_{k-1})$ is selected for \mathbf{b} , where the mean value 0_{k-1} is the $(k-1)$ -dimensional zero vector and the

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variance is the $(k-1)$ -dimensional identity matrix I_{k-1} multiplied by a step increment t_{Δ} of the Brown movement and the scaling factor ε .

- 5 An alternative method of determining the predictor is first to determine a normally distributed random number vector in the $(m+n+k)$ -dimensional variable space and then to project said vector into the $(k-1)$ -dimensional tangential plane $T_Z M$.
- 10 After this, in step 105, the predictor is projected with the aid of a corrector method, which is, for example, a numerical Newton method, onto the manifold of the pareto-optimal points. In this way a new valid operating point is determined on the manifold of the pareto-optimal points.
- 15 The steps 103, 104 and 105 are repeated iteratively, with the operating point determined in the preceding iteration step being used as the starting point for calculating a new valid operating point.
- 20 In step 106 a check is made to determine whether an abort criterion has been met, in other words whether, for example, a predetermined number of iterations have been performed or a predetermined time limit has been reached. If this is not the
- 25 case, a return is made to step 103 and the next iteration is performed. This is continued until the abort criterion is met.
- Once the abort criterion has been met in step 106, in a next step 107 the set of determined pareto-optimal points is
- 30 restricted to those points in which the weighting factors α_i are positive.

From these pareto-optimal points, in a final step 108, the user selects an efficient operating point of the technical

system appropriate to his/her requirements and the technical system is designed using this efficient operating point.

Figure 2 shows a two-dimensional graphical representation of the predictor-corrector method used in the invention.

In Figure 2, z^i designates a pareto-optimal point on the submanifold M , with this point having been obtained in the i -th iteration step of the method. In order to determine a new pareto-optimal point, the tangential plane $T_{z^i}M$ to the submanifold M is first determined in the point z^i . The tangential plane is indicated by means of dashed lines in Figure 2. In the next step 104, a predictor point y^{i+1} is then determined using normally distributed random numbers in the tangential plane $T_{z^i}M$. In the following corrector step 105, which can be, for example, a Newton method, the new pareto-optimal point z^{i+1} is determined. The method is then continued, with the pareto-optimal point z^{i+1} being used as the starting point for new predictor step.

Figure 3 relates to a variation of the method according to the invention, wherein if predictors with negative weighting factors α_i occur, a reflection is performed in order to determine a new predictor with positive α_i . Figure 3 shows this projection step being performed in a three-dimensional representation.

Figure 3 depicts a case in which, starting from a pareto-optimal point z , a predictor y_{neg} is determined which has a negative α_i . This is illustrated graphically in that the section between the point z and the point y penetrates the tangential plane T_zM in the point S . In this case the point S in turn lies on a subplane of the tangential plane T_zM , for

the points of which the coordinate α_i has the value zero. In order to perform the reflection, the point of intersection S is determined first. This can be done using a projection operator which projects the α_i component from a parameter representation of the straight line running through the points z and y . After the point S has been determined, the vector x_{neg} running between S and y can now be determined. This vector is then dissected into the tangential component t to the subplane and into a normal component n . Thus, $t = x_{neg} - n$ applies to the tangential component. The reflection step is then performed, with the new reflected vector x_{neu} having the same tangential component t as the old vector x_{neg} and the normal component corresponding to the normal component n of the old vector x_{neg} with the sign reversed. The new vector is therefore $x_{neu} = t - n = (x_{neg} - n) - n = x_{neg} - 2n$. There thus results a new predictor y_{neu} which was reflected at the tangential plane $T_z M$. $y_{neu} = S + x_{neu}$ applies to the new predictor point y_{neu} . The above described reflection method increases the numerical efficiency of the method since the generation of points with negative weighting factors α_i is avoided and consequently the technical computing resources are used more efficiently.

Fig. 4 shows a processor unit PRZE for performing the method according to the invention. The processor unit PRZE comprises a processor CPU, a memory MEM and an input/output interface IOS which is used in a different way via an interface IFC: An output is made visible on a monitor MON via a graphical interface and/or output on a printer PRT. An input is made via a mouse MAS or a keyboard TAST. The processor unit PRZE also has a data bus BUS which provides the connection from a memory MEM, the processor CPU and the input/output interface IOS. Additional components such as, for example, additional memory,

data storage (hard disk) or scanner can also be connected to the data bus BUS.

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